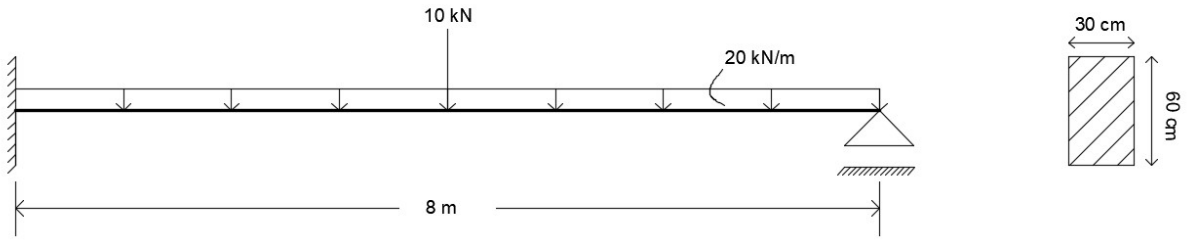
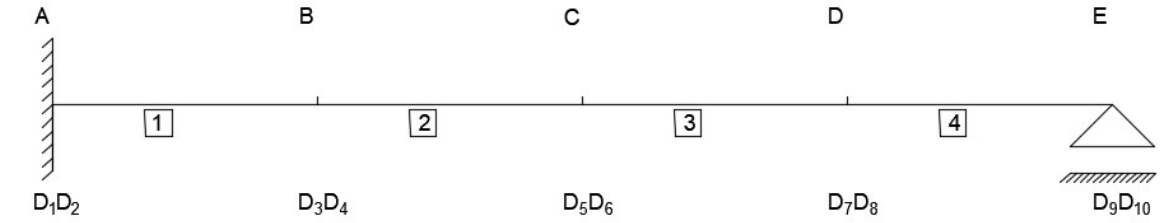
SORU:

Aşağıdaki sistemi dört eş parçaya bölerek sonlu elemanlar yöntemi ile moment dağılımını elde ediniz.

(Malzeme beton sınıfı C30)



CEVAP:



$$\{d\}=\begin{matrix} w_A \\ \theta_A \\ w_B \\ \theta_B \end{matrix}=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} \qquad \{D\}^T=[D_1 \quad D_2 \quad D_3 \quad D_4 \quad D_5 \quad D_6 \quad D_7 \quad D_8 \quad D_9 \quad D_{10}]$$

$w= a_1+a_2x+a_3x^2+a_4x^3$ $\qquad \theta= a_2+2a_3x+3a_4x^2$ $\qquad w=[\Phi(x)]\{a\}$

$\{d\}=[A]\{a\} \rightarrow \{a\}=[A]^{-1}\{d\} \rightarrow [N]= [\Phi(x)][A]^{-1}$

$$[A]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L & L^2 & L^3 \\ 0 & 1 & 2L & 3L^2 \end{bmatrix} \quad ; \quad [A]^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{2}{L} & \frac{3}{L^2} & -\frac{1}{L} \\ \frac{2}{L^3} & \frac{1}{L^2} & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix}$$

$[N]= [\Phi(x)][A]^{-1} = [N_1 \quad N_2 \quad N_3 \quad N_4]= [1 \quad x \quad x^2 \quad x^3]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{2}{L} & \frac{3}{L^2} & -\frac{1}{L} \\ \frac{2}{L^3} & \frac{1}{L^2} & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix}$

$\rightarrow \quad N_1=1-\frac{3}{L^2}x^2+\frac{2}{L^3}x^3 \quad ; \quad N_2=L\left[\frac{x}{L}-\frac{2x^2}{L^2}+\frac{x^3}{L^3}\right] \quad ; \quad N_3=\frac{3}{L^2}x^2-\frac{2}{L^3}x^3 \quad ;$

$N_4= L\left[-\frac{x^2}{L^2}+\frac{x^3}{L^3}\right]$

$\delta\prod^e=\{d\}^T[[k_e]\{d\}-[q_e]]$

$[k_e]=\int\{\varepsilon\}^T[D]\{\varepsilon\}dx \quad ; \quad \{\varepsilon\}=-\frac{\partial^2[N]}{\partial x^2}\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$

$[k_e]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$

<u>Çevirme matrisleri;</u>

$[C_{e1}]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$[C_{e2}]=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

$[C_{e3}]=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

$[C_{e4}]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$[K_e]=[C_e]^T[k_e][C_e] \quad \rightarrow$

$$[K_e]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 & 0 & 0 & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -12L & -6L & 24 & 0 & -12 & 6L & 0 & 0 & 0 & 0 \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & -6L & 24 & 0 & -12 & 6L & 0 & 0 \\ 0 & 0 & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -12 & -6L & 24 & 0 & -12 & 6L \\ 0 & 0 & 0 & 0 & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 0 & 0 & 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

<u>Yük vektörünün hesaplanması;</u>

$\{Q_e\}=[C_e]^T\{q_e\}$ →

$$\{Q_e\}= q\left(\begin{bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ L/2 \\ L^2/12 \\ L/2 \\ -L'/12 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{bmatrix}\right)+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} qL/2 \\ qL^2/12 \\ qL \\ 0 \\ qL+P \\ 0 \\ qL \\ 0 \\ qL/2 \\ -qL^2/12 \end{bmatrix}$$

$[K]\{D\}=\{Q\}$

$D_1=D_2=D_9=0$

$$\frac{EI}{L^3}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 & -12 & 6L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8L^2 & -6L & 2L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & -6L & 24 & 0 & -12 & 6L & 0 & 0 \\ 0 & 0 & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -12 & -6L & 24 & 0 & 0 & 6L \\ 0 & 0 & 0 & 0 & 6L & 2L^2 & 0 & 8L^2 & 0 & 2L^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6L & 2L^2 & 0 & 4L^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ 0 \\ D_{10} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ qL \\ 0 \\ qL+P \\ 0 \\ qL \\ 0 \\ 0 \\ -qL^2/12 \end{bmatrix}$$

$D_1=0$ $D_4=0.00094$ $D_7=0.0023$ $D_{10}=-0.00135$

$D_2=0$ $D_5=0.00274$ $D_8=-0.000775$

$D_3=0.00128$ $D_6=0.00034$ $D_9=0$

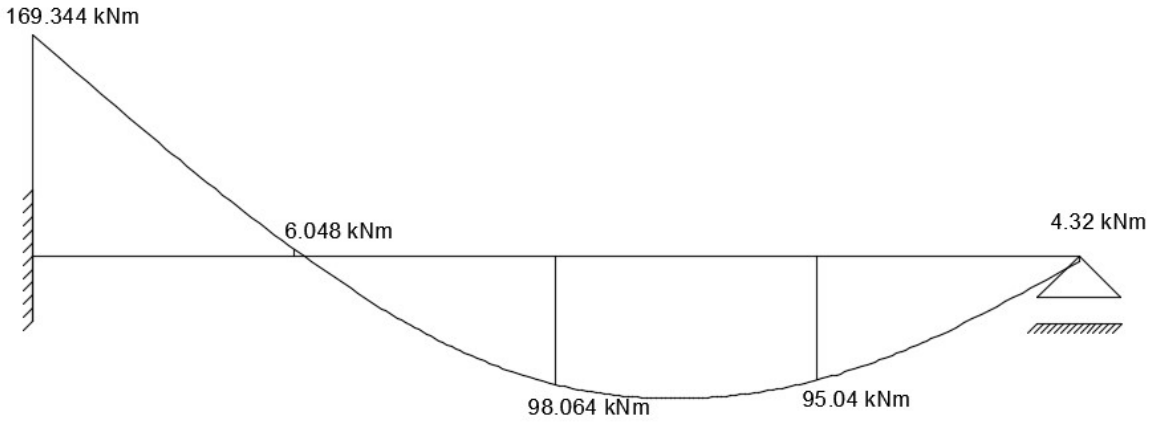
M_i=-EIw'' →

M_A^1=-169.344 kNm M_C^3=97.632 kNm

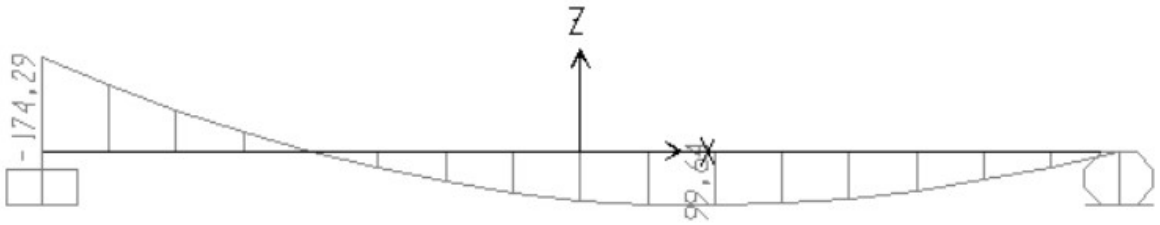
M_B^1=6.912 kNm M_D^3=95.04 kNm

M_B^2=5.184 kNm M_D^4=95.04 kNm

M_C^2=98.496 kNm M_E^4=4.32 kNm



<u>SAP2000 VERİLERİ;</u>



M_0=-174.29 kNm

M_2= 0 kNm

M_4= 92.85 kNm

M_6= 86.43 kNm

M_8= 0 kNm